一百學年度第一學期

電機工程學系

微分方程同儕輔導手冊

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1. 說明「學習成效診斷與預警」之進行方式與成效。

本課程之「學習成效診斷與預警」進行方式，是透過檢查小考考試成績後，篩選出學習成效不佳的學生，再由所安排之輔導人員(本系研究生)進行成績追蹤，並要求其接受課後補救教學。經過輔導跟教學後成績有明顯提升。

2.說明「課程同儕輔導」之規劃與實行過程

我們規畫一個禮拜 1 次的輔導課程,每次課程至多不超過 3 個小時,提供同學在上完課之後,有任何問題可以在這些時間進行提問,並在每次小考、期中和期末之前則會再另外安排額外的時間,給同學複習提問。

3.說明修課學生最普遍遭遇的「學習瓶頸」與「迷失概念」。

經歸納發現，修習本課程學生常遭遇之問題如下：

1.死背公式，不了解公式的原理及應用方法。

2.死背推導過程，卻不知微分方程方法的適用時機。

4.可提供未來修課學生參考之「常見問題類型」與「解題方向」

解題方向:因變數變更法(換 y)、自變數變更法(換 x)。

解題方向:利用分佈積分工是有效縮短計算量。

常見問題:二階變係數 ODE。

常見問題: power Series 計算錯誤以己同整合併時往往不能化到最簡表示

5.可提供未來教師教學時需特別留意或加強的單元名稱

1.線性獨立與線性相依的判斷方式經常搞錯。

2. power Series 計算量龐大需要多加注意解題技巧和同整

3.Legendre’s Equation 的解題技巧需要熟悉了解,不可以全靠背誦。

6.參與課業輔導之學生人數統計,以及學生接受課業輔導前後的成績分布與改變差異。

期中分數分布:

分數	100	99-90	89-80	79-70	69-60	59-50	40 以下
人數	1 人	4 人	9 人	10 人	13 人	10 人	18 人

第二次小考分數分布:

分數	100	99-90	89-80	79-70	69-60	59-50	40 以下
人數	7 人	9 人	9 人	15 人	10 人	5 人	9 人

經過輔導後,成績有明顯變好!!

7.授課教師所提供學生的「練習或小考題目」、「延伸閱讀」、「補充資料」等相關資料。

微分方程練習題目

1-1

Solve the ODE by integration or by remembering a differentiation formula

1.$y' + 2\sin 2\pi x = 0$

6. $y'' = -y$

(a)Verify that y is a solution of the ODE.(b)Determine from y the particular solution of the IVP.(c)Graph the solution of the IVP.

13. $y' = y - y^2, y = \frac{1}{1+ce^{-x}}, y(0) = 0.25$, ,

1-2

Graph a direction field(by a CAS or by hand). In the field graph several solution curves by hand, particularly those passing through the given points (x,y)

8.$y' = -2xy, (0, \frac{1}{2}), (0,1), (0,2)$

1-3

Find a general solution. Show the steps of derivation. Check your answer by substitution

1. $y^3 y' + x^3 = 0$

6. $y' = e^{2x-1} y^2$

10. $xy' = x + y$ (set $y/x = u$)

Solve the IVP. Show the steps of derivation, beginning with the general solution.

12. $y' = 1 + 4y^2, y(1) = 0$

16. $y' = (x + y - 2)^2, y(0) = 2$ (set $v = x + y - 2$),

1-4

Test for exactness. If exact, solve. If not, use an integrating factor as given or obtained by inspection or by the theorems in the text. Also, if an initial condition is given, find the corresponding particular solution.

2. $x^3 dx + y^3 dy = 0$

8. $e^x(\cos y\ dx - \sin y\ dy) = 0$

12. $(2xy\,dx + dy)e^{x^2} = 0, y(0) = 2$

14. $(a + 1)y\,dx + (b + 1)x\,dy = 0$, $y(1) = 1$, $F = x^a y^b$

1-5

Find the general solution. If an initial condition is given, find also the corresponding particular solution and graph or sketch it. (show the details of your work)

4. $y' = 2y - 4x$

10. $y' \cos x + (3y - 1) \sec x = 0 \, , y\left(\frac{1}{4}\pi\right) = \frac{4}{3}$

12. $xy' + 4y = 8x^4 \, , y(1) = 2$

Using a method of this section or separating variables, find the general solution. If an initial condition is given, find also the particular solution and sketch or graph it.

22. $y' + y = y^2 \, , y(0) = -\frac{1}{3}$

26. $y' = (\tan y)/(x-1)$, $y(0) = \frac{1}{2}\pi$

28. $2xyy' + (x-1)y^2 = x^2 e^x \; (\text{set } y^2 = z)$

2-1

Reduce to first order and solve, showing each step in detail

4. $2xy'' = 3y'$

6. $xy'' + 2y' + xy = 0$, $y_1 = (\cos x)/x$

10. $y'' + (1 + {}^1\!/_y) y'^2 = 0$

(a)Verify that the given functions are linearly independent and form a basis of solutions of the given ODE (b) Solve the IVP.

15. $y'' + 9y = 0$, $y(0) = 2$; $y'(0) = -1$, $\cos 3x, \sin 3x$

16. $y'' + 2y' + y = 0$, $y(0) = 2$, $y'(0) = -1$, e^{-x} , xe^{-x}

18. $x^2y'' - xy' + y = 0$, $y(1) = 1; y'(1) = 2$, x , $x\ln(x)$

2-2

Find a general solution

2. $y'' + 36y = 0$

6. $10y'' - 32y' + 25.6y = 0$

10. $100y'' + 240y' + (196\pi^2 + 144)y = 0$

14. $y'' + 2k^2y' + k^4y = 0$

Find an ODE ($y'' + ay' + by = 0$, for the given basis)

18. $\cos 2\pi x$, $\sin 2\pi x$

Solve the IVP. Check that your answer satisfies the ODE as well as the initial conditions. Show the details of your work.

24. $y'' - 2y' - 3y = 0$, $y(-1) = e$, $y'(-1) = {}^{-e}/_{4}$
28. $6y'' - y' - y = 0$, $y(0) = -0.5$, $y'(0) = 1.25$
30. $9y'' - 30y' + 25y = 0$, $y(0) = 3.3$, $y'(0) = 10.0$

Are the following functions linearly independent on the given interval?

32. e^{ax} , e^{-ax} , $x > 0$
34. $\ln x$, $\ln(x^3)$, $x > 1$
36. $e^{-x} \cos \frac{1}{4} x$, 0 , $-1 \leq x \leq 1$

2-3

Apply the given operator to the given functions. Show all steps in detail.

2. $D - 3I$; $3x^2 + 3x$, $3e^{3x}$, $\cos 4x - \sin 4x$
5. $(D + I)(D - 2I)$; e^{4x} , xe^{4x} , e^{-2x}

Factor as in the text and solve.

6. $(D^2 + 4.00D + 3.36I)y = 0$
8. $(D^2 + 3I)y = 0$
12. $(D^2 + 3.0D + 2.5I)y = 0$

2-5

Find a real general solution. Show the details of your work.

2. $x^2y'' - 2y = 0$
8. $(x^2D^2 - 3xD + 4I)y = 0$
10. $(x^2D^2 - xD + 5I)y = 0$

Solve the solution. Show the details of your work.

12. $x^2y'' - 4xy' + 6y = 0$, $y(1) = 0.4$, $y'(1) = 0$
16. $(x^2D^2 - 3xD + 4I)y = 0$, $y(1) = -\pi$, $y'(1) = 2\pi$
18. $(9x^2D^2 + 3xD + I)y = 0$, $y(1) = 1$, $y'(1) = 0$

2-6

Find the Wronskian. Show linear independence by using quotients and confirm it by $W(y_1, y_2) = y_1{y'}_2 - y_2{y'}_1$

2. $e^{4.0x}$, $e^{-1.5x}$
4. $2x$, ${}^{1}/_{(4x)}$
6. $e^{-x} \cos \omega x$, $e^{-x} \sin \omega x$

(a) Find a second-order homogeneous linear ODE for which the given functions are solutions. (b) Show linear independence by the Wronskian. (c) Solve the initial value problem.

10. x^{m_1} , x^{m_2} , $y(1) = -2$, $y'(1) = 2m_1 - 4m_2$
12. x^2 , $x^2 \ln x$, $y(1) = 4$, $y'(1) = 6$
14. $e^{-kx}\cos \pi x$, $e^{-kx}\sin \pi x$, $y(0) = 1$, $y'(0) = -k - \pi$

2-7

Find a general solution. State with rule you are using. Show each step of your work.

1. $y'' + 5y' + 6y = 2e^{-x}$
4. $y'' - 4y = 8\cos \pi x$
6. $y'' + y' + (\pi^2 + \frac{1}{4})y = e^{-x/2}\sin \pi x$
8. $(3D^2 + 27I)y = 3\cos x + \cos 3x$

Solve the initial value problem. State which rule you are using. Show each step of your calculation in detail.

12. $y'' + 4y = -12\sin 2x$, $y(0) = 1.8$, $y'(0) = 5.0$
14. $y'' + 6y' + 9y = e^{-x}\cos 2x$, $y(0) = 1$, $y'(0) = -1$
16. $(D^2 - 2D)y = 6e^{2x} - 4e^{-2x}$, $y(0) = -1$, $y'(0) = 6$
18. $(D^2 + 2D + 10I)y = 17\sin x - 37\sin 3x$, $y(0) = 6.6$, $y'(0) = -2.2$

2-10

Solve the given nonhomogeneous linear ODE by variation of parameters or undetermined coefficients. Show the details of your work.

2. $y'' + 9y = \csc 3x$
3. $x^2y'' - xy' - 3y = x^2$
4. $y'' - 4y' + 5y = e^{2x}\csc x$
6. $(D^2 + 6D + 9I)y = 16e^{-3x}/(x^2 + 1)$
10. $(D^2 + 2D + 2I)y = 4e^{-x}\sec^3 x$
13. $(x^2D^2 + xD - 9I)y = 48x^5$

3-1

To get a feel for higher order ODEs, show that the given functions are solutions and form a basis on any interval.

2. e^x , e^{-x} , $e^{x/2}$, $2y''' - y'' - 2y' + y = 0$
6. 1 , x^2 , x^4 , $x^2y''' - 3xy'' + 3y' = 0$

Are the given functions linearly independent or dependent on the half-axis x≥0 ? Give reason.

8. x^2 , $1/x^2$, 0

10. e^{2x} , xe^{2x} , x^2e^{2x}
12. $\sin^2 x$, $\cos^2 x$, $\cos 2x$
14. $\cos^2 x$, $\sin^2 x$, 2π

3-2

Solve the given ODE. Show the details of your work.

2. $y^{iv} + 2y'' + y = 0$
4. $(D^3 - D^2 - D + I)y = 0$

Solve the IVP by CAS, giving a general solution and the particular solution and its graph.

10. $y^{iv} + 4y = 0$, $y(0) = \frac{1}{2}$, $y'(0) = -\frac{3}{2}$, $y''(0) = \frac{5}{2}$, $y'''(0) = -\frac{7}{2}$
12. $y^{v} - 5y''' + 4y' = 0$, $y(0) = 3$, $y'(0) = -5$, $y''(0) = 11$, $y'''(0) = -23$, $y^{iv}(0) = 47$

3-3

Solve the following ODEs, showing the details of your work

2. $y''' + 2y'' - y' - 2y = 1 - 4x^3$
4. $(D^3 + 3D^2 - 5D - 39I)y = -300 \cos x$
6. $(D^3 + 4D)y = \sin x$

Solve the given IVP, showing the details of your work.

8. $y^{iv} - 5y'' + 4y = 10e^{-3x}$, $y(0) = 1$, $y'(0) = 0$, $y''(0) = 0$, $y'''(0) = 0$
10. $x^3y''' + xy' - y = x^2$, $y(1) = 1$, $y'(1) = 3$, $y''(1) = 14$
12. $(D^3 - 2D^2 - 9D + 18I)y = e^{2x}$, $y(0) = 4.5$, $y'(0) = 8.8$, $y''(0) = 17.2$

5-1

Determine the radius of convergence. Show the details of your work

2.

$$\sum_{m=0}^{\infty}(m+1)mx^m$$

4.

$$\sum_{m=0}^{\infty}\frac{x^{2m+1}}{(2m+1)!}$$

5

$$\sum_{m=0}^{\infty}\left(\frac{2}{3}\right)^m x^{2m}$$

Apply the power series method. Show the details.

6. $(1+x)y' = 2y$

8. $xy' - 4y = k$ (k a constant)

Find a power series solution in Powers of x. Show the details.

10. $y'' - y' + xy = 0$

12. $(1-x^2)y'' - 2xy' + 2y = 0$

14. $y'' - 4xy' + (4x^2 - 2)y = 0$

Solve the initial value problem by a power series.Find the value of the sum s (5 digits)at x_1

16. $y' + 4y = 1\,, y(0) = 1.25\,, x_1 = 0.2$

18. $(1-x^2)y'' - 2xy' + 30y = 0\,, y(0) = 0\,, y'(0) = 1.875\,, x_1 = 0.5$

5-2

1. Legendre functions for $n = 0$. show that (6) with $n = 0$ gives $P_0(x) = 1$ and (7)gives $\left(\text{use } \ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 + \cdots\right)$, $y_2(x) = x + \frac{1}{3}x^3 + \frac{1}{5}x^5 + \cdots = \frac{1}{2}\ln\frac{1+x}{1-x}$ verify this by solving (1) with $n = 0$, setting $z = y'$ and separating variables

2. Legendre functions for $n = 1$. Show that (7) with $n = 1$ gives $y_2(x) = P_1(x)$ and (6) gives , $y_1 = 1 - x^2 - \frac{1}{3}x^4 - \frac{1}{5}x^6 - \cdots = 1 - \frac{1}{2}x\ln\frac{1+x}{1-x}$

5-3

Find a basis of solutions by the Frobenius method. Try to identify the series as expansions of known functions. Show the details of your work.

2. $(x+1)^2y'' + (x+1)y' - y = 0$
6. $xy'' + 2x^3y' + (x^2-2)y = 0$
12. $x^2y'' + 6xy' + (4x^2+6)y = 0$

5-4

Find a general solution in terms of J_v and J_{-v} or indicate when this is not possible. Use the indicated substitutions. Show the details of your work.

2. $x^2y'' + xy' + \left(x^2 - \frac{1}{9}\right)y = 0$

6. $x^2y'' + \left(\frac{3}{16} + x\right)y = 0 \quad (y = 2u\sqrt{x}\ ,\ \sqrt{x} = z)$

10. $x^2y'' + (1-2v)xy' + v^2(x^{2v} + 1 - v^2)y = 0\ ,\ (y = x^v u\ ,\ x^v = z)$

Use the powerful formulas to do Probs. Show the details of your work.

19.

Derivatives. Show that $J'_0(x) = -J_1(x), J'_1(x) = J_0(x) - J_1(x)/x\ , J'_2(x) = \frac{1}{2}[J_1(x) - J_3(x)]$.

22.

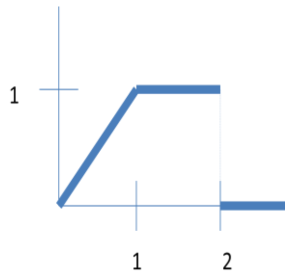
Basic integral formulas. Show that $\int x^{-v} J_{v+1}(x)dx = -x^{-v}J_v(x) + c\ , \int J_{v+1}(x)dx = \int J_{v-1}(x)dx - 2J_v(x)$

6-1

Find the transform. Show the details of your work. Assume that a, b ,θ,ω are constants.

2. $(a-bt)^2$
4. $\cos^2\omega t$
6. $e^{-t}\sinh 4t$

12.



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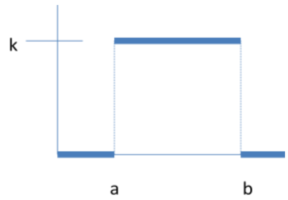

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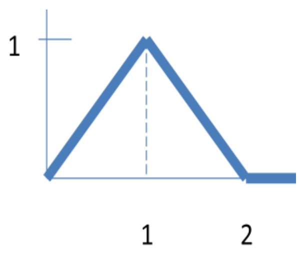

Given $F(s) = \pounds(f)$, find $f(t)$. a , b , L , n are constants. Show the details of your work

26. $\frac{5s+1}{s^2-25}$

32. $\frac{1}{(s+a)(s+b)}$

34. $ke^{-at}\cos\omega t$

38. $\frac{6}{(s+1)^3}$

42. $\frac{a_0}{s+1} + \frac{a_1}{(s+1)^2} + \frac{a_2}{(s+1)^3}$

44. $\frac{a(s+k)+b\pi}{(s+k)^2+\pi^2}$

6-2

Solve the IVPs by the Laplace transform.

4. $y'' + 9y = 10e^{-t}\ ,\ y(0) = 0\ ,y'(0) = 0$
6. $y'' - 6y' + 5y = 29\cos 2t\ ,y(0) = 3.2\ ,\ y'(0) = 6.2$
10. $y'' + 0.04y = 0.02t^2\ ,\ y(0) = -25\ ,\ y'(0) = 0$

Solve the shifted data IVPs by the Laplace transform. Show the details.

12. $y'' + 2y' - 3y = 0\ ,\ y(2) = -3\ ,\ y'(2) = -5$

26. $\frac{1}{s^4-s^2}$

6-3

Find its transform. Show the details of your work

2. $t(0 < t < 2)$
6. $\sin\pi t(2 < t < 4)$
10. $\sinh t(0 < t < 2)$
14. $4(e^{-2s} - 2e^{-5s})/s$

Using the Laplace transform and showing the details

20. $y'' + 10y' + 24y = 144t^2\ ,\ y(0) = \frac{19}{12}\ ,y'(0) = -5$
24. $y'' + 3y' + 2y = 1$ if $0 < t < 1$ *and* 0 *if* $t > 1\ ; y(0) = 0\ ,\ y'(0) = 0$

6-4

Find the solution of the IVP. Show the details.

4. $y'' + 16y = 4\delta(t - 3\pi),\ y(0) = 2\ , y'(0) = 0$

8. $y'' + 3y' + 2y = 10(\sin t + \delta(t - 1)),\ y(0) = 1\ , y'(0) = -1$

12. $y'' + 2y' + 5y = 25t - 100\delta(t - \pi),\ y(0) = -2\ , y'(0) = 5$

6-5

2. $1 * \sin\omega t$

4. $(\cos\omega t) * (\cos\omega t)$

Solve by the Laplace transform, showing the details.

8. $y(t) + 4\int_0^t y(\tau)(t - \tau)d\tau = 2t$

10. $y(t) - \int_0^t y(\tau)\sin 2(t - \tau)d\tau = \sin 2t$

14. $y(t) - \int_0^t y(\tau)(t - \tau)d\tau = 2 - \frac{1}{2}t^2$

Showing details

18. $\dfrac{1}{(s-a)^2}$

20. $\dfrac{9}{s(s+3)}$

24. $\dfrac{240}{(s^2+1)(s^2+25)}$

6-6

Showing the details of your work, find $\pounds(f)$ if $f(t)$ equals

2. $3t\sinh 4t$

6. $t^2 \sin 3t$

10. $t^n e^{kt}$

Showing the details, find $f(t)$ if $\pounds(f)$ equals:

14. $\dfrac{s}{(s^2+16)^2}$

18. $\operatorname{arccot}\dfrac{s}{\pi}$

8.課程輔導小老師的工作心得整理

透過這次的輔導,可以說是複習自己的以前工程數學的微分方程部份,也更加了解到一般學生對於工數學習態度與解題常遇到的困難.基本上都是計算上的小問題才過來詢問,工數上的數學原理,和老師上課所說原理,學生大部分都能吸收,只有少數觀念較複雜的問題才需要寫版書計算過程給學生們看,計算部份不外乎是微積分部份比較弱,可能是大一時微積分的底子沒有打好,這些部分有待學生自己加強計算工數的時間,才能大幅縮短計算時間.

9.受輔學生對此項方案的感想與意見分析。

透過助教的安排的輔導時間內我們可以有效的學習到解題的技巧並且增進自身計算能力.

10.授課老師對「學生學習成效預警與課程同儕輔導」制度的期許與建議

學生在學習上難免會碰到問題,可是老師不一定能完全解答到每個學生的問題,所以有了這個輔導課,能更幫助學習